

LINEAR PROGRAMMING

Question 1

Formulate the dual for the following linear program:

Maximise : $100x_1 + 90x_2 + 40x_3 + 60x_4$

Subject to

$$6x_1 + 4x_2 + 8x_3 + 4x_4 \leq 140$$

$$10x_1 + 10x_2 + 2x_3 + 6x_4 \leq 120$$

$$10x_1 + 12x_2 + 6x_3 + 2x_4 \leq 50$$

$$x_1, x_2, x_3, x_4 \geq 0$$

(Only formulation is required. Please do not solve.)

(6 Marks)(June, 2009)

Answer

Dual:

Minimise $140u_1 + 120u_2 + 50u_3$

S.T. $6u_1 + 10u_2 + 10u_3 \geq 100$

$$4u_1 + 10u_2 + 12u_3 \geq 90$$

$$8u_1 + 2u_2 + 6u_3 \geq 40$$

$$4u_1 + 6u_2 + 2u_3 \geq 60$$

$$u_1, u_2, u_3, u_4 \geq 0$$

Question 2

The following is a linear programming problem. You are required to set up the initial simplex tableau. (Please do not attempt further iterations or solution):

Maximise

$$100x_1 = 80x_2$$

Subject to

$$3x_1 + 5x_2 \leq 150$$

$$x_2 \leq 20$$

$$8x_1 + 5x_2 \leq 300$$

$$x_1 + x_2 \geq 25$$

$$x_1, x_2 \geq 0$$

(6 Marks)(Nov., 2009)

Answer

Under the usual notations where

S1, S2, S3 are stock Variables,

A4 = the artificial variable

S4 = Surplus Variable

We have,

$$\text{Max. } Z = 100x_1 + 80x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4 - M A_4.$$

S.t.

$$3x_1 + 5x_2 + S_1 = 150$$

$$x_2 + S_2 = 20$$

$$8x_1 + 5x_2 + S_3 = 300$$

$$x_1 + x_2 - S_4 + A_4 = 25$$

		x_1	x_2	S_1	S_2	S_3	S_4	A_4		
Basis	C_j	100	80	0	0	0	0	-M		
S_1	0	3	5	1	0	0	0	0	150	✓
S_2	0	0	1	0	1	0	0	0	20	✓
S_3	0	8	5	0	0	1	0	0	300	✓
A_4	-M	1	1	0	0	0	-1	1	25	✓
Z_j		-M	-M	0	0	0	M	-M	-25M	✓
$C_j - Z_j$		100+M	80+M	0	0	0	-M	0		✓

Question 3

A firm makes two products X and Y, and has a total production capacity of 16 tonnes per day. X and Y are requiring the same production capacity. The firm has a permanent contract to supply at least 3 tonnes of X and 6 tonnes of Y per day to another company. Each tonne of X

require 14 machine hours of production time and each tonne of Y requires 20 machine hours of production time. the daily maximum possible number of machine hours is 280. All the firm's output can be sold, and the profit made is ₹ 20 per tonne of X and ₹ 25 per tonne of Y.

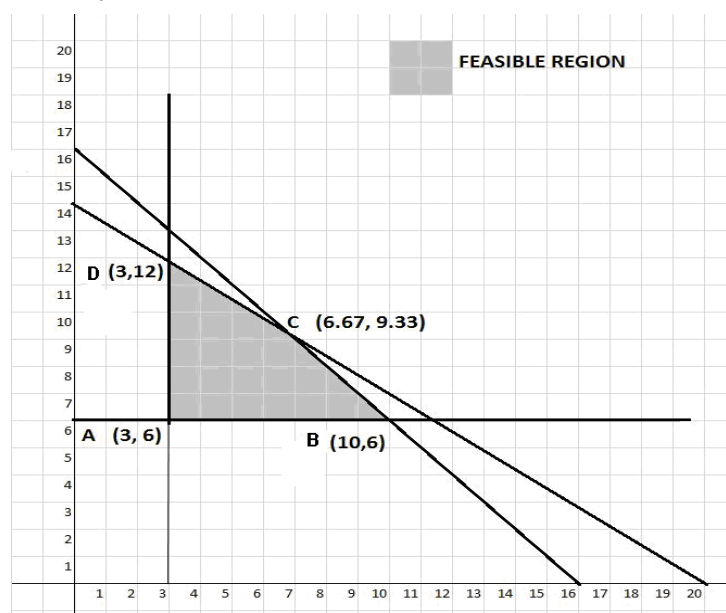
Required:

Formulate a linear programme to determine the production schedule for maximum profit by using graphical approach and calculate the optimal product mix and profit.

(6 Marks)(Nov., 2010)

Answer

Maximise Z $20x + 25y$
 Subject to $x + y \leq 16$
 $x \geq 3$
 $y \geq 6$
 $14x + 20y \leq 280$
 $x, y > 0$



Z =	20x +	25y	Total Contribution
Point	X	Y	
A	3	6	210
B	10	6	350

C	6.67	9.33	367- Optimal
D	3	12	360

The maximum value of objective function $Z = 370$ occurs at extreme point C (6.67, 9.33).

Hence company should produce $x_1 = 6.67$ tonnes of product X and $x_2 = 9.33$ tones of prod Y in order to yield a maximum profit of Rs. 367.

Question 4

The following matrix gives the unit cost of transporting a product from production plants P_1 , P_2 and P_3 to destinations. D_1 , D_2 and D_3 . Plants P_1 , P_2 and P_3 have a maximum production of 65, 24 and 111 units respectively and destinations D_1 , D_2 and D_3 must receive at least 60, 65 and 75 units respectively:

To \ From	D_1	D_2	D_3	Supply
P_1	400	600	800	65
P_2	1,000	1,200	1,400	24
P_3	500	900	700	111
Demand	60	65	75	200

You are required to formulate the above as a linear programming problem. (Only formulation is needed. Please do not solve). (6 Marks) (Nov., 2008)

Answer

Let p_{ij} be the variable to denote the number of units of product from the i th plant to the j th destination, so that

P_1d_1 = transport from plant P_1 to D_1

P_2d_2 = transport from plant P_2 to D_2 etc.

Objective function

$$\text{Minimize } z = 400 p_{1d_1} + 600 p_{1d_2} + 800 p_{1d_3} + 1000 p_{2d_1} + 1200 p_{2d_2} + 1400 p_{2d_3} \\ + 500 p_{3d_1} + 900 p_{3d_2} + 700 p_{3d_3}.$$

Subject to:

$$\begin{aligned} p_{1d_1} + p_{1d_2} + p_{1d_3} &\leq 65 \\ p_{2d_1} + p_{2d_2} + p_{2d_3} &\leq 24 \\ p_{3d_1} + p_{3d_2} + p_{3d_3} &\leq 111 \end{aligned} \quad \begin{array}{c} \uparrow \\ \uparrow \\ \uparrow \end{array} \text{ (Plant constraints)}$$

and

$$\begin{aligned}
 p_1 d_1 + p_2 d_1 + p_3 d_1 &= 60 \\
 p_1 d_2 + p_2 d_2 + p_3 d_2 &= 65 \\
 p_1 d_3 + p_2 d_3 + p_3 d_3 &= 75
 \end{aligned}
 \begin{array}{l}
 \uparrow \\
 \text{(destination constraints)} \\
 \uparrow \\
 \text{p}
 \end{array}$$

all $p_i d_j \geq 0$